Water density

CENTRO NACIONAL DE METROLOGÍA

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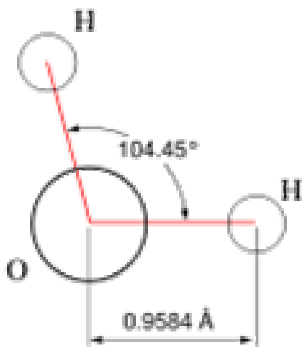
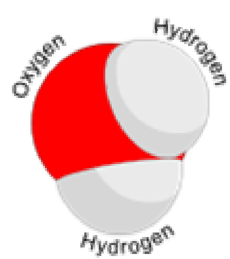
Water is a tasteless, odourless substance that is essential to all known forms of life and is known as the universal solvent. It appears colourless to the naked eye in small quantities. It covers nearly 70% of Earth's surface.

Water molecule is formed by two atoms of hydrogen and one of oxygen.

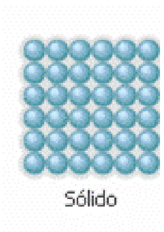


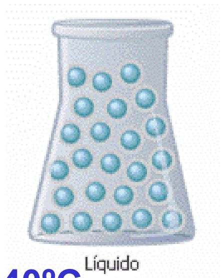


Density of water

States of matter of water

Solid **liquid** **Gas**





0ºC **40ºC** **100ºC**

Temperature

Water density formula by M. Tanaka et al.

IAPWS Formulation, International Association for the Properties of Water and Steam (IAPWS95)

Water density formula by M. Tanaka et al.

The density of the water of de-aerated SMOW at a pressure of 101 325 Pa and at temperature t expressed in terms of the ITS-90 is given as

$$\rho_w = a_5\left[1 - \frac{(t+a_1)^2(t+a_2)}{a_3(t+a_4)}\right]$$

$a_1/{}^\circ C =$ -3,983 035 ± 0,000 67

$a_2/{}^\circ C =$ 301,797

$a_3/{}^\circ C =$ 522 528,9

$a_4/{}^\circ C =$ 69,348 81

$a_5/(\mathbf{kg\,m^{-3}}) =$ 999,974 950 ±0,000 84

The relative uncertainty of this formula is $1\text{x}10^{-6}$

VSMOW, or Vienna Standard Mean Ocean Water,

is an isotopic water standard defined in 1968 by the International Atomic Energy Agency. VSMOW serves as a reference standard for comparing hydrogen and oxygen isotope ratios, mostly in water samples.

Isotopic abundance

Most users of water as a density standard rely on tap water instead of SMOW.
a_5 must be changed in this case, (Girard and Menaché)

$$a_5' = a_5 + 0{,}233\delta_{18} + 0{,}016\,6\delta_D$$

Where a_5' is the modified parameter, and the quantities δ_{18} and δ_D are defined by the relations,

$$\delta_{18} = \left[r_{18}(\text{sample}) / r_{18}(\text{SMOW}) - 1\right] x 10^3$$

$$\delta_D = \left[r_D(\text{sample}) / r_D(\text{SMOW}) - 1\right] x 10^3$$

Where r_{18} is amount of substance ratio $\left[{}^{18}O\right] / \left[{}^{16}O\right]$ and r_D is the amount of substance ratio $[D]/[H]$ Chappuis found next value for tap water

$$a_5' = 999{,}972 \text{ kg m}^{-3}$$

Dissolved air

The density of water has been given under the assumption that the water is air-free.

N. Bignell has determined the difference in density between air-free and air-saturated water.

Between 0ºC and 25ºC this difference is described by the following formula,

$$C_{dg} / \left(\mathrm{kg\,m^{-3}}\right) = s_0 + s_1 t$$

$$s_0 / \left(10^{-3}\ \mathrm{kg\,m^{-3}}\right) = -4{,}612$$

$$s_1 / \left(10^{-3}\ \mathrm{kg\,m^{-3}\,{}^{\circ}C^{-1}}\right) = 0{,}106$$

Compressibility

The density of the aerated water has been given at pressure p of 101 325 Pa (one atmosphere).

The compressibility factor for the water density used at different pressure is

$$f_C = \left[1 + \left(k_0 + k_1 t + k_2 t^2\right)\Delta p\right]$$

$$\Delta p / Pa = p / Pa - 101\ 325\ Pa$$

$$k_0 / \left(10^{-11} Pa^{-1}\right) = 50{,}74$$

$$k_1 / \left(10^{-11} Pa^{-1}\,{}^{\circ}C^{-1}\right) = -0{,}326$$

$$k_2 / \left(10^{-11} Pa^{-1}\,{}^{\circ}C^{-2}\right) = 0{,}004\,16$$

Numerical example

A sample at the following conditions,

Temperature	unc. (k=1)	Pressure	unc. (k=1)
ºC	ºC	Pa	Pa
20	0,05	81 000	10

The isotopic composition of the sample of water is,

	value	unc. (k=1)
$\delta^{18}O$ ($^{0}/_{00}$)	-9,88	0,10
δD ($^{0}/_{00}$)	-75,0	1,3

The value of a_5 evaluated with the isotopic composition of the water sample

$$a_5' = a_5 + 0{,}233\delta_{18} + 0{,}016\,6\delta_D = $$ **999,971 40 kg m^{-3} ± 3x10^{-5}** kg m^{-3}

The water density before to corrections due to dissolved air and pressure,

$$\rho_w{}' = a_5'\left[1 - \frac{(t+a_1)^2(t+a_2)}{a_3(t+a_4)}\right] = $$ **998,203 2 kg m^{-3}**

Correction due to dissolved gasses in the water sample,

$$C_{dg} = s_0 + s_1 t =$$ **-2,49x10^{-3} kg m^{-3}**

Correction factor due to pressure difference (reference pressure 101 325 Pa and the measured pressure)

$$f_C = \left[1 + \left(k_0 + k_1 t + k_2 t^2\right)\Delta p\right] =$$ **0,999 990 7**

The corrected value of water density is,

$$\rho_w = \rho'_w \cdot f_c + C_{dg} = \quad \mathbf{998{,}191\ kg\ m^{-3}}$$

The water density uncertainty evaluated by GUM method is,

$$u(\rho_w) = \sqrt{\sum_{i=1}^{N} \left[c_i u(x_i)\right]^2} = \sqrt{\sum_{i=1}^{N} \left(\frac{\partial \rho_w}{\partial x_i}\right)^2 u(x_i)^2}$$

$$u(\rho_w) = \mathbf{\pm\ 0{,}010\ kg\ m^{-3}}$$

Table of budget of uncertainty

input quant.	Sens. Coeff.		std unc.		Contribution	variance	contribution %
temperature	-2.06E-01	kg/(m3 ºC)	0.05	ºC	-0.01031786	1.06E-04	**99.07**
pressure	4.58E-07	kg/(m3 Pa)	10	Pa	4.5802E-06	2.10E-11	**0.00**
fitting	1		0.001	kg/m3	0.001	1.00E-06	**0.93**
a5	0.99823175 1		3.15E-05	kg/m3	3.15E-05	9.91E-10	**0.00**